

**Standard 1: Number and Computation**

**NINTH AND TENTH GRADES**

**Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.**

**Benchmark 1: Number Sense – The student demonstrates number sense for real numbers and algebraic expressions in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. knows, explains, and uses equivalent representations for real numbers and algebraic expressions including integers, fractions, decimals, percents, ratios; rational number bases with integer exponents; rational numbers written in scientific notation; absolute value; time; and money (2.4.K1a) (\$), e.g., <math>-4/2 = (-2)</math>; <math>a^{(-2)} b^{(3)} = b^3/a^2</math>.</li> <li>2. compares and orders real numbers and/or algebraic expressions and explains the relative magnitude between them (2.4.K1a) (\$), e.g., will <math>(5n)^2</math> always, sometimes, or never be larger than <math>5n</math>? The student might respond with <math>(5n)^2</math> is greater than <math>5n</math> if <math>n &gt; 1</math> and <math>(5n)^2</math> is smaller than <math>5n</math> if <math>0 &lt; n &lt; 1</math>.</li> <li>3. knows and explains what happens to the product or quotient when a real number is multiplied or divided by (2.4.K1a):               <ol style="list-style-type: none"> <li>a. a rational number greater than zero and less than one,</li> <li>b. a rational number greater than one,</li> <li>c. a rational number less than zero.</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. generates and/or solves real-world problems using equivalent representations of real numbers and algebraic expressions (2.4.A1a) (\$), e.g., a math classroom needs 30 books and 15 calculators. If B represents the cost of a book and C represents the cost of a calculator, generate two different expressions to represent the cost of books and calculators for 9 math classrooms.</li> <li>2. determines whether or not solutions to real-world problems using real numbers and algebraic expressions are reasonable (2.4.A1a) (\$), e.g., in January, a business gave its employees a 10% raise. The following year, due to the sluggish economy, the employees decided to take a 10% reduction in their salary. Is it reasonable to say they are now making the same wage they made prior to the 10% raise?</li> </ol>

9/10-1  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes: Number sense** refers to one's ability to reason with numbers and to work with numbers in a flexible way. The ability to compute mentally, to estimate based on understanding of number relationships and magnitudes, and to judge reasonableness of answers are all involved in number sense.

At this grade level, real numbers include positive and negative numbers and very large numbers (one billion) and very small numbers (one-billionth). **Relative magnitude** refers to the size relationship one number has with another – is it much larger, much smaller, close, or about the same? For example, using the numbers 219, 264, and 457, answer questions such as

Which two are closest? Why?

Which is closest to 300? To 250?

About how far apart are 219 and 500? 5,000?

If these are 'big numbers,' what are small numbers? Numbers about the same? Numbers that make these seem small?

(Elementary and Middle School Mathematics, John A. Van de Walle, Addison Wesley Longman, Inc., 1998)

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-2  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 1: Number and Computation**

**NINTH AND TENTH GRADES**

**Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.**

**Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of the real number system; recognizes, applies, and explains their properties, and extends these properties to algebraic expressions.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. explains and illustrates the relationship between the subsets of the real number system [natural (counting) numbers, whole numbers, integers, rational numbers, irrational numbers] using mathematical models (2.4.K1a), e.g., number lines or Venn diagrams.</li> <li>2. identifies all the subsets of the real number system [natural (counting) numbers, whole numbers, integers, rational numbers, irrational numbers] to which a given number belongs (2.4.K1m).</li> <li>3. ▲ names, uses, and describes these properties with the real number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$):               <ol style="list-style-type: none"> <li>a. commutative (<math>a + b = b + a</math> and <math>ab = ba</math>), associative [<math>a + (b + c) = (a + b) + c</math> and <math>a(bc) = (ab)c</math>], distributive [<math>a(b + c) = ab + ac</math>], and substitution properties (if <math>a = 2</math>, then <math>3a = 3 \times 2 = 6</math>);</li> <li>b. identity properties for addition and multiplication and inverse properties of addition and multiplication (additive identity: <math>a + 0 = a</math>, multiplicative identity: <math>a \cdot 1 = a</math>, additive inverse: <math>+5 + -5 = 0</math>, multiplicative inverse: <math>8 \times 1/8 = 1</math>);</li> <li>c. symmetric property of equality (if <math>a = b</math>, then <math>b = a</math>);</li> <li>d. addition and multiplication properties of equality (if <math>a = b</math>, then <math>a + c = b + c</math> and if <math>a = b</math>, then <math>ac = bc</math>) and inequalities (if <math>a &gt; b</math>, then <math>a + c &gt; b + c</math> and if <math>a &gt; b</math>, and <math>c &gt; 0</math> then <math>ac &gt; bc</math>);</li> <li>e. zero product property (if <math>ab = 0</math>, then <math>a = 0</math> and/or <math>b = 0</math>).</li> </ol> </li> <li>4. uses and describes these properties with the real number system (2.4.K1a) (\$):               <ol style="list-style-type: none"> <li>a. transitive property (if <math>a = b</math> and <math>b = c</math>, then <math>a = c</math>),</li> <li>b. reflexive property (<math>a = a</math>).</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. generates and/or solves real-world problems with real numbers using the concepts of these properties to explain reasoning (2.4.A1a) (\$):               <ol style="list-style-type: none"> <li>a. commutative, associative, distributive, and substitution properties, e.g., the chorus is sponsoring a trip to an amusement park. They need to purchase 15 adult tickets at \$6 each and 15 student tickets at \$4 each. How much money will the chorus need for tickets? Solve this problem two ways.</li> <li>b. identity and inverse properties of addition and multiplication, e.g., the purchase price (P) of a series EE Savings Bond is found by the formula <math>\frac{1}{2} F = P</math> where F is the face value of the bond. Use the formula to find the face value of a savings bond purchased for \$500.</li> <li>c. symmetric property of equality, e.g., Sam took a \$15 check to the bank and received a \$10 bill and a \$5 bill. Later Sam took a \$10 bill and a \$5 bill to the bank and received a check for \$15. \$ addition and multiplication properties of equality, e.g., the total price for the purchase of three shirts is \$62.54 including tax. If the tax is \$3.89, what is the cost of one shirt, if all shirts cost the same?</li> <li>d. zero product property, e.g., Jenny was thinking of two numbers. Jenny said that the product of the two numbers was 0. What could you deduce from this statement? Explain your reasoning.</li> <li>e. zero product property, e.g., Jenny was thinking of two numbers. Jenny said that the product of the two numbers was 0. What could you deduce from this statement? Explain your reasoning.</li> </ol> </li> </ol>

9/10-3  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

	<p>2. analyzes and evaluates the advantages and disadvantages of using integers, whole numbers, fractions (including mixed numbers), decimals or irrational numbers and their rational approximations in solving a given real-world problem (2.4.A1a) (\$), e.g., a store sells CDs for \$12.99 each. Knowing that the sales tax is 7%, Marie estimates the cost of a CD plus tax to be \$14.30. She selects nine CDs. The clerk tells Marie her bill is \$157.18. How can Marie explain to the clerk she has been overcharged?</p>
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**Teacher Notes:** From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

Property of a number: 8 is divisible by 2.

Property of a geometric shape: Each of the four sides of a square is of the same length.

Property of an operation: Addition is commutative. For all numbers  $x$  and  $y$ ,  $x + y = y + x$ .

Property of an equation: For all numbers  $a$ ,  $b$ , and  $c$ , if  $a = b$ , then  $a + c = b + c$ .

Property of an inequality: For all numbers  $a$ ,  $b$ , and  $c$ , if  $a > b$ , then  $a - c > b - c$ .

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-4  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 1: Number and Computation**

**NINTH AND TENTH GRADES**

**Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.**

**Benchmark 3: Estimation – The student uses computational estimation with real numbers in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>estimates real number quantities using various computational methods including mental math, paper and pencil, concrete objects, and/or appropriate technology (2.4.K1a) (\$).</li> <li>uses various estimation strategies and explains how they were used to estimate real number quantities and algebraic expressions (2.4.K1a) (\$).</li> <li>knows and explains why a decimal representation of an irrational number is an approximate value(2.4.K1a).</li> <li>knows and explains between which two consecutive integers an irrational number lies (2.4.K1a).</li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>▲ adjusts original rational number estimate of a real-world problem based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., estimate how long it takes to walk from here to there; time how long it takes to take five steps and adjust your estimate.</li> <li>estimates to check whether or not the result of a real-world problem using real numbers and/or algebraic expressions is reasonable and makes predictions based on the information (2.4.A1a) (\$), e.g., if you have a \$4,000 debt on a credit card and the minimum of \$30 is paid per month, is it reasonable to pay off the debt in 10 years?</li> <li>determines if a real-world problem calls for an exact or approximate answer and performs the appropriate computation using various computational strategies including mental math, paper and pencil, concrete objects, and/or appropriate technology (2.4.A1a) (\$), e.g., do you need an exact or an approximate answer in calculating the area of the walls to determine the number of rolls of wallpaper needed to paper a room? What would you do if you were wallpapering 2 rooms?</li> <li>explains the impact of estimation on the result of a real-world problem (underestimate, overestimate, range of estimates) (2.4.A1a) (\$), e.g., if the weight of 25 pieces of paper was measured as 530.6 grams, what would the weight of 2,000 pieces of paper equal to the nearest gram? If the student were to estimate the weight of one piece of paper as about 20 grams and then multiply this by 2,000 rather than multiply the weight of 25 pieces of paper by 80; the answer would differ by about 2,400 grams. In general, multiplying or dividing by a rounded number will cause greater discrepancies than rounding after multiplying or dividing.</li> </ol>

9/10-5  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes: Estimate**, as a verb, means to make an educated guess based on information in a problem or to give an answer close to the exact number. Estimation is used when an exact answer is not needed, as in many real-life situations for which “ballpark” computations are acceptable. Good number sense enables one to estimate a quantity, estimate a measure, or estimate an answer.

**Estimation** serves as an important companion to computation. It provides a tool for judging the reasonableness of computational methods including mental math, paper and pencil, concrete objects, and appropriate technology. However, being able to compute does not automatically lead to an ability to estimate or judge reasonableness of answers. Frequent modeling by the teacher helps students develop a range of estimation strategies. Students should be encouraged to frequently explain their thinking as they estimate. As with exact computation, sharing estimation strategies allows students access to others’ thinking and provides opportunities for class discussion. Identifying the estimation strategy by name is not critical; however, explaining the thinking behind the strategy to make a valid estimation is important. (Principles and Standards for School Mathematics, NCTM, 2000)

**Mental math** and **estimation** are distinct but related mathematical skills. Proficiency in mental math contributes to increased skill in estimation. In order for students to become more familiar with estimation, teachers should introduce estimation with examples where rounded or estimated numbers are used. Emphasis should be placed on real-world examples where only estimation is required, e.g., About how many hours do you sleep a night? Using the language of estimation is important, so students begin to realize that a variety of estimates (answers) are possible. In addition, when students are taught specific estimation strategies, they develop mental math and estimation skills easier. Estimation strategies include front-end with adjustment, compatible “nice” numbers, clustering, special numbers, truncation, or simulation.

**Mathematical models** such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **(\$)**. The National Standards in Personal Finance are included in the Appendix.

9/10-6  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 1: Number and Computation**

**NINTH AND TENTH GRADES**

**Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.**

**Benchmark 4: Computation – The student models, performs, and explains computation with real numbers and polynomials in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (<b>\$</b>).</li> <li>2. performs and explains these computational procedures (2.4.K1a):               <ol style="list-style-type: none"> <li>a. <b>N</b> addition, subtraction, multiplication, and division using the order of operations</li> <li>b. multiplication or division to find (<b>\$</b>):                   <ol style="list-style-type: none"> <li>i. a percent of a number, e.g., what is 0.5% of 10?</li> <li>ii. percent of increase and decrease, e.g., a college raises its tuition from \$1,320 per year to \$1,425 per year. What percent is the change in tuition?</li> <li>iii. percent one number is of another number, e.g., 89 is what percent of 82?</li> <li>iv. a number when a percent of the number is given, e.g., 80 is 32% of what number?</li> </ol> </li> <li>c. manipulation of variable quantities within an equation or inequality (2.4.K1d), e.g., <math>5x - 3y = 20</math> could be written as <math>5x - 20 = 3y</math> or <math>5x(2x + 3) = 8</math> could be written as <math>8/(5x) = 2x + 3</math>;</li> <li>d. simplification of radical expressions (without rationalizing denominators) including square roots of perfect square monomials and cube roots of perfect cubic monomials;</li> <li>e. simplification or evaluation of real numbers and algebraic monomial expressions raised to a whole number power and algebraic binomial expressions squared or cubed;</li> <li>f. simplification of products and quotients of real number and algebraic monomial expressions using the properties of exponents;</li> </ol> </li> </ol>	<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>1. generates and/or solves multi-step real-world problems with real numbers and algebraic expressions using computational procedures (addition, subtraction, multiplication, division, roots, and powers excluding logarithms), and mathematical concepts with (<b>\$</b>):               <ol style="list-style-type: none"> <li>a. <b>▲</b> applications from business, chemistry, and physics that involve addition, subtraction, multiplication, division, squares, and square roots when the formulae are given as part of the problem and variables are defined (2.4.A1a) (<b>\$</b>), e.g., given <math>F = ma</math>, where <math>F</math> = force in newtons, <math>m</math> = mass in kilograms, <math>a</math> = acceleration in meters per second squared. Find the acceleration if a force of 20 newtons is applied to a mass of 3 kilograms.</li> <li>b. <b>▲</b> volume and surface area given the measurement formulas of rectangular solids and cylinders (2.4.A1f), e.g., a silo has a diameter of 8 feet and a height of 20 feet. How many cubic feet of grain can it store?</li> <li>c. probabilities (2.4.A1h), e.g., if the probability of getting a defective light bulb is 2%, and you buy 150 light bulbs, how many would you expect to be defective?</li> <li>d. <b>▲ ■</b> application of percents (2.4.A1a), e.g., given the formula <math>A = P(1 + \frac{r}{n})^{nt}</math>, <math>A</math> = amount, <math>P</math> = principal, <math>r</math> = annual interest, <math>n</math> = number of compounding periods per year, <math>t</math> = number of years. If \$1,000 is placed in a savings account with a 6% annual interest rate and is compounded semiannually, how much money will be in the account at the end of 2 years?</li> </ol> </li> </ol>

9/10-7  
January 31, 2004

**▲ – Assessed Indicator**

**■ – Assessed Indicator on the Optional Response Assessment**

**N – Noncalculator**

**(\$) – Financial Literacy**

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

<p>g. matrix addition (\$), e.g., when computing (with one operation) a building's expenses (data) monthly, a matrix is created to include each of the different expenses; then at the end of the year, each type of expense for the building is totaled;</p> <p>h. scalar-matrix multiplication (\$), e.g., if a matrix is created with everyone's salary in it, and everyone gets a 10% raise in pay; to find the new salary, the matrix would be multiplied by 1.1.</p> <p>3. finds prime factors, greatest common factor, multiples, and the least common multiple of algebraic expressions (2.4.K1b).</p>	<p>e. simple exponential growth and decay (excluding logarithms) and economics (2.4.A1a) (\$), e.g., a population of cells doubles every 20 years. If there are 20 cells to start with, how long will it take for there to be more than 150 cells? <i>or</i> If the radiation level is now 400 and it decays by <math>\frac{1}{2}</math> or its half-life is 8 hours, how long will it take for the radiation level to be below an acceptable level of 5?</p>
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9/10-8  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

**Teacher Notes: Efficiency and accuracy** means that students are able to compute with fluency. Students increase their understanding and skill in addition, subtraction, multiplication, and division by understanding the relationships between addition and subtraction, addition and multiplication, multiplication and division, and subtraction and division. Students learn basic number combinations and develop strategies for computing that makes sense to them. Through class discussions, students can compare the ease of use and ease of explanation of various strategies. In some cases, their strategies for computing will be close to conventional algorithms; in other cases, they will be quite different. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and these invented approaches often can be used with efficiency and accuracy. (Principles and Standards for School Mathematics, NCTM, 2000)

The definition of computation is finding the standard representation for a number. For example,  $6 + 6$ ,  $4 \times 3$ ,  $17 - 5$ , and  $24 \div 2$  are all representations for the standard representation of 12. **Mental math** is mentally finding the standard representation for a number – calculating in your head instead of calculating using paper and pencil or technology. One of the main reasons for teaching mental math is to help students determine if a computed/calculated answer is reasonable; in other words, using mental math to estimate to see if the answer makes sense. Students develop mental math skills easier when they are taught specific strategies. Mental math strategies include counting on, doubling, repeated doubling, halving, making tens, multiplying by powers of tens, dividing with tens, finding fractional parts, thinking money, and using compatible “nice” numbers.

**Mathematical models** such as concrete objects, pictures, diagrams, Venn diagrams, number lines, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

**Technology** is changing mathematics and its uses. The use of technology including calculators and computers is an important part of growing up in a complex society. It is not only necessary to estimate appropriate answers accurately when required, but also it is also important to have a good understanding of the underlying concepts in order to know when to apply the appropriate procedure. Technology does not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation. However, dividing a 5-digit number by a 2-digit number is appropriate with the exception of dividing by 10, 100, or 1,000 and simple multiples of each.

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9/10-9  
January 31, 2004

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N – Noncalculator

(**\$**) – Financial Literacy

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**Standard 2: Algebra**

**NINTH AND TENTH GRADES**

**Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.**

**Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains the general rule of a pattern in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. identifies, states, and continues the following patterns using various formats including numeric (list or table), algebraic (symbolic notation), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written               <ol style="list-style-type: none"> <li>a. arithmetic and geometric sequences using real numbers and/or exponents (2.4.K1a); e.g., radioactive half-lives;</li> <li>b. patterns using geometric figures (2.4.K1h);</li> <li>c. algebraic patterns including consecutive number patterns or equations of functions, e.g., <math>n</math>, <math>n + 1</math>, <math>n + 2</math>, ... or <math>f(n) = 2n - 1</math> (2.4.K1c,e);</li> <li>d. special patterns (2.4.K1a), e.g., Pascal's triangle and the Fibonacci sequence.</li> </ol> </li> <li>2. generates and explains a pattern (2.4.K1f).</li> <li>3. classify sequences as arithmetic, geometric, or neither.</li> <li>4. defines (2.4.K1a):               <ol style="list-style-type: none"> <li>a. a recursive or explicit formula for arithmetic sequences and finds any particular term,</li> <li>b. a recursive or explicit formula for geometric sequences and finds any particular term.</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. recognizes the same general pattern presented in different representations [numeric (list or table), visual (picture, table, or graph), and written] (2.4.A1i) (<b>\$</b>).</li> <li>2. solves real-world problems with arithmetic or geometric sequences by using the explicit equation of the sequence (2.4.K1c) (<b>\$</b>), e.g., an arithmetic sequence: A brick wall is 3 feet high and the owners want to build it higher. If the builders can lay 2 feet every hour, how long will it take to raise it to a height of 20 feet? or a geometric sequence: A savings program can double your money every 12 years. If you place \$100 in the program, how many years will it take to have over \$1,000?</li> </ol>

9/10-10  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes:** A fundamental component in the development of classification, number, and problem solving skills is inventing, discovering, and describing patterns. Patterns pervade all of mathematics and much of nature. All **patterns** are either repeating or growing or a variation of either or both. Translating a pattern from one medium to another to find two patterns that are alike, even though they are made with different materials, is important so students can see the relationships that are critical to repeating patterns. With growing patterns, students not only extend patterns, but also look for a generalization or an algebraic relationship that will tell them what the pattern will be at any point along the way.

Working with **patterns** is an important process in the development of mathematical thinking. Patterns can be based on geometric attributes (shapes, regions, angles); measurement attributes (color, texture, length, weight, volume, number); relational attributes (proportion, sequence, functions); and affective attributes (values, likes, dislikes, familiarity, heritage, culture). (Learning to Teach Mathematics, Randall J. Souviney, Macmillan Publishing Company, 1994)

In the pattern that begins with 3, 5, 7, and 9; the explicit rule is  $2n + 1$  and the recursive rule is add 2 to the previous term. Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

This process (working with patterns) can be used to develop or deepen understandings of important concepts in number theory, rational numbers, measurement, geometry, probability, and functions. Working with patterns provides opportunities for students to recognize, describe, extend, develop, and explain.

**Number theory** is the study of the properties of the counting numbers (positive integers), their relationships, ways to represent them, and patterns among them. Number theory includes the concepts of odd and even numbers, factors and multiples, primes and composites, and greatest common factor and least common multiple, and sequences.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (**\$**). The National Standards in Personal Finance are included in the Appendix.

9/10-11  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 2: Algebra**

**NINTH AND TENTH GRADES**

**Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.**

**Benchmark 2: Variables, Equations, and Inequalities – The student uses variables, symbols, real numbers, and algebraic expressions to solve equations and inequalities in variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. knows and explains the use of variables as parameters for a specific variable situation (2.4.K1f), e.g., the m and b in <math>y = mx + b</math> or the h, k, and r in <math>(x - h)^2 + (y - k)^2 = r^2</math>.</li> <li>2. manipulates variable quantities within an equation or inequality (2.4.K1e), e.g., <math>5x - 3y = 20</math> could be written as <math>5x - 20 = 3y</math> or <math>5x(2x + 3) = 8</math> could be written as <math>8/(5x) = 2x + 3</math>.</li> <li>3. solves (2.4.K1d) (\$):             <ol style="list-style-type: none"> <li>a. <b>N</b> linear equations and inequalities both analytically and graphically;</li> <li>b. quadratic equations with integer solutions (may be solved by trial and error, graphing, quadratic formula, or factoring);</li> <li>c. <b>▲N</b> systems of linear equations with two unknowns using integer coefficients and constants;</li> <li>d. radical equations with no more than one inverse operation around the radical expression;</li> <li>e. equations where the solution to a rational equation can be simplified as a linear equation with a nonzero denominator, e.g., <math>\frac{3}{(x + 2)} = \frac{5}{(x - 3)}</math>.</li> <li>f. equations and inequalities with absolute value quantities containing one variable with a special emphasis on using a number line and the concept of absolute value.</li> <li>g. exponential equations with the same base without the aid of a calculator or computer, e.g., <math>3^{x+2} = 3^5</math>.</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. represents real-world problems using variables, symbols, expressions, equations, inequalities, and simple systems of linear equations (2.4.A1c-e) (\$).</li> <li>2. represents and/or solves real-world problems with (2.4.A1c) (\$):             <ol style="list-style-type: none"> <li>a. <b>▲N</b> linear equations and inequalities both analytically and graphically, e.g., tickets for a school play are \$5 for adults and \$3 for students. You need to sell at least \$65 in tickets. Give an inequality and a graph that represents this situation and three possible solutions.</li> <li>b. quadratic equations with integer solutions (may be solved by trial and error, graphing, quadratic formula, or factoring), e.g., a fence is to be built onto an existing fence. The three sides will be built with 2,000 meters of fencing. To maximize the rectangular area, what should be the dimensions of the fence?</li> <li>c. systems of linear equations with two unknowns, e.g., when comparing two cellular telephone plans, Plan A costs \$10 per month and \$.10 per minute and Plan B costs \$12 per month and \$.07 per minute. The problem is represented by Plan A = <math>.10x + 10</math> and Plan B = <math>.07x + 12</math> where x is the number of minutes.</li> <li>d. radical equations with no more than one inverse operation around the radical expression, e.g., a square rug with an area of 200 square feet is 4 feet shorter than a room. What is the length of the room?</li> <li>e. a rational equation where the solution can be simplified as a linear equation with a nonzero denominator, e.g., John is 2 feet taller than Fred. John's shadow is 6 feet in length and Fred's shadow is 4 feet in length. How tall is Fred?</li> </ol> </li> </ol>

9/10-12  
January 31, 2004

**▲ – Assessed Indicator**

**■ – Assessed Indicator on the Optional Response Assessment**

**N – Noncalculator**

**(\$)** – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

3. explains the mathematical reasoning that was used to solve a real-world problem using equations and inequalities and analyzes the advantages and disadvantages of various strategies that may have been used to solve the problem (2.4.A1c).

**Teacher Notes:** One of the most powerful ways we have of expressing the regularities found in mathematics is with variables. Variables enable us to use mathematical symbolism as a tool to think and help better understand mathematical ideas in the same way physical objects and drawings are used. But if variables are to be included with these other “thinker toys,” it is important to help students develop an understanding of the various ways they are used. A **variable** is a symbol that can stand for any one of a set of numbers or other objects. Although correct, this simple-sounding definition has a variety of interpretations, depending on how the variables are used.

Meanings of variables change with the way they are used. Usiskin (The ideas of Algebra, K-12, pp.8-19, NCEM, 1988) identified three uses of variables that are commonly encountered in school mathematics:

1. *As a specific unknown.* In the early grades, this is the use found in equations such as  $8 + \square = 12$ . Later, we see exercises such as this: If  $3x + 2 = 4x - 1$ , solve for  $x$ .
2. *As a pattern generalizer.* Variables are used in statements that are true for all numbers. For example,  $a \times b = b \times a$  for all real numbers. Formulas such as  $A = L \times W$  also show a pattern.
3. *As quantities that vary in joint variation.* *Joint variation* occurs when change in one variable determines a change in another. In  $y = 3x + 5$ , as  $x$  changes or varies, so does  $y$ . Formulas are also an example of joint variation. In  $C = 2\pi r$ , the radius, changes, so does  $C$ , the circumference. (Elementary and Middle School Mathematics, John A. Van de Walle, Addison Wesley Longman, Inc., 1988)d

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-13  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 2: Algebra**

**NINTH AND TENTH GRADES**

**Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.**

**Benchmark 3: Functions – The student analyzes functions in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>evaluates and analyzes functions using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or other appropriate technology (2.4.K1a,d-f).</li> <li>matches equations and graphs of constant and linear functions and quadratic functions limited to <math>y = ax^2 + c</math> (2.4.K1d,f).</li> <li>determines whether a graph, list of ordered pairs, table of values, or rule represents a function (2.4.K1e-f).</li> <li>determines x- and y-intercepts and maximum and minimum values of the portion of the graph that is shown on a coordinate plane (2.4.K1f).</li> <li>identifies domain and range of:             <ol style="list-style-type: none"> <li>relationships given the graph or table (2.4.K1e-f),</li> <li>linear, constant, and quadratic functions given the equation(s) (2.4.K1d).</li> </ol> </li> <li>▲ recognizes how changes in the constant and/or slope within a linear function changes the appearance of a graph (2.4.K1f) (\$).</li> <li>uses function notation.</li> <li>evaluates function(s) given a specific domain (\$).</li> <li>describes the difference between independent and dependent variables and identifies independent and dependent variables (\$).</li> </ol>	<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>translates between the numerical, graphical, and symbolic representations of functions (2.4.A1c-e) (\$).</li> <li>▲ ■ interprets the meaning of the x- and y- intercepts, slope, and/or points on and off the line on a graph in the context of a real-world situation (2.4.A1e) (\$), e.g., the graph below represents a tank full of water being emptied. What does the y-intercept represent? What does the x-intercept represent? What is the rate at which it is emptying? What does the point (2, 25) represent in this situation? What does the point (2,30) represent in this situation?  <b>The Water Tank</b></li> </ol> <ol style="list-style-type: none"> <li>analyzes (2.4.A1c-e):             <ol style="list-style-type: none"> <li>the effects of parameter changes (scale changes or restricted domains) on the appearance of a function's graph,</li> <li>how changes in the constants and/or slope within a linear function affects the appearance of a graph,</li> <li>how changes in the constants and/or coefficients within a quadratic function in the form of <math>y = ax^2 + c</math> affects the appearance of a graph.</li> </ol> </li> </ol>

9/10-14  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes: Functions** are relationships or rules in which each member of one set is paired with one, and only one, member of another set (an ordered pair). The concept of function can be introduced using function machines. Any number put in the machine will be changed according to some rule. A record of inputs and corresponding outputs can be maintained in a two-column format. Function tables, input/output machines, and T-tables may be used interchangeably and serve the same purpose.

Function concepts should be developed from **growing patterns**. Each term in a number sequence is related to its position in the sequence – the functional relationship. The pattern – 4, 7, 10, 13, 16, 19, and so on – is an arithmetic sequence *with a difference of 3*. The pattern could be described as *add 3* meaning that 3 must be added to the previous term to find the next. This pattern is explained by using the recursive definition for a sequence. The recursive definition for a sequence is a statement or a set of statements that explains how each successive term in the sequence is obtained from the previous term(s).

In the pattern 1, 4, 9, 16, 25, ..., 225; there is *no common difference*. This sequence is not arithmetic or geometric (no common ratio between geometric terms). Neither is it a combination of the two; however, there is a pattern and the missing terms between 25 and 225 can be found. To find the term value, square the number of the term. The next missing terms would be 36, 49, 64, 81, 100, 121, and 144. This pattern is explained by using the explicit formula for a sequence. The explicit formula for a sequence defines a rule for finding each term in the number sequence related to its position in the sequence. In other words, to find the term value, square the number of the term – the 5<sup>th</sup> term is 5<sup>2</sup>, the 8<sup>th</sup> term is 8<sup>2</sup>, ...

Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

In the Cartesian Coordinate System, the expression  $x = 3$  cannot represent a function. When you graph all points where  $x = 3$  you get a vertical line, so more than one  $y$ -value exists for the  $x$ -value  $x = 3$ . By definition, a function must have only one output value for any given input. For more information, consult a reference book under the topic, vertical line test.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in Personal Finance identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-15  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 2: Algebra**

**NINTH AND TENTH GRADES**

**Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.**

**Benchmark 4: Models – The student develops and uses mathematical models to represent and justify mathematical relationships found in a variety of situations involving tenth grade knowledge and skills.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. knows, explains, and uses mathematical models to represent and explain mathematical concepts, procedures, and relationships. Mathematical models include:               <ol style="list-style-type: none"> <li>a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, and mathematical relationships and to solve equations (1.1.K1-3, 1.2.K1, 1.2.K3-4, 1.3.K1-4, 1.4.K1, 1.4.K2a-b, 2.1.K1a, 2.1.K1d, 2.1.K2, 2.2.K4, 2.3.K1, 3.2.K1-3, 3.2.K6, 3.3.K1-4, 4.2.K3-4) (\$);</li> <li>b. factor trees to model least common multiple, greatest common factor, and prime factorization (1.4.K3);</li> <li>c. algebraic expressions to model relationships between two successive numbers in a sequence or other numerical patterns (2.1.K1c);</li> <li>d. equations and inequalities to model numerical and geometric relationships (1.4.K2c, 2.2.K3, 2.3.K1-2, 3.2.K7) (\$);</li> <li>e. function tables to model numerical and algebraic relationships (2.1.K1c, 2.2.K2, 2.3.K1, 2.3.K3, 2.3.K5) (\$);</li> <li>f. coordinate planes to model relationships between ordered pairs and equations and inequalities and linear and quadratic functions (2.2.K1, 2.3.K1-6, 3.4.K1-8) (\$);</li> <li>g. constructions to model geometric theorems and properties (3.1.K2, 3.1.K6);</li> <li>h. two- and three-dimensional geometric models (geoboards, dot paper, coordinate plane, nets, or solids) and real-world objects to model perimeter, area, volume, and surface area, properties of two- and three-dimensional figures, and isometric views of three-dimensional figures (2.1.K1b, 3.1.K1-8, 3.2.K1, 3.2.K4-5, 3.3.K1-4);</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include:               <ol style="list-style-type: none"> <li>a. process models (concrete objects, pictures, diagrams, flowcharts, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, mathematical relationships, and problem situations and to solve equations (1.1.K1, 1.2.A1-2, 1.3.A1-4, 1.4.A1a, 1.4.A1d-e, 3.1.A1-3, 3.2.A1-3, 3.3.A2, 3.3.A4, 3.4.A2, 4.2.A1a-b) (\$);</li> <li>b. algebraic expressions to model relationships between two successive numbers in a sequence or other numerical patterns;</li> <li>c. equations and inequalities to model numerical and geometric relationships (2.1.A2, 2.2.A1-3, 2.3.A1) (\$);</li> <li>d. function tables to model numerical and algebraic relationships (2.3.A1, 2.3.A3, 3.4.A2) (\$);</li> <li>e. coordinate planes to model relationships between ordered pairs and equations and inequalities and linear and quadratic functions (2.2.A1, 2.3.A1-3, 3.4.A1-2, 3.4.A4) (\$);</li> <li>f. two- and three-dimensional geometric models (geoboards, dot paper, coordinate plane, nets, or solids) and real-world objects to model perimeter, area, volume, and surface area, properties of two- and three-dimensional figures and isometric views of three-dimensional figures (3.3.A1, 4.2.A1c);</li> <li>g. scale drawings to model large and small real-world objects (3.3.A3, 3.4.A3);</li> <li>h. geometric models (spinners, targets, or number cubes), process models (coins, pictures, or diagrams), and tree diagrams to model probability (1.4.A1c, 4.2.A1, 4.2.A3);</li> </ol> </li> </ol>

9/10-16  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$ ) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

<ul style="list-style-type: none"> <li>i. scale drawings to model large and small real-world objects;</li> <li>j. Pascal's Triangle to model binomial expansion and probability;</li> <li>k. geometric models (spinners, targets, or number cubes), process models (concrete objects, pictures, diagrams, or coins), and tree diagrams to model probability (4.1.K1-3);</li> <li>l. frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single and double stem-and-leaf plots, scatter plots, box-and-whisker plots, histograms, and matrices to organize and display data (4.2.K1, 4.2.K5-6) (<b>\$</b>);</li> <li>m. Venn diagrams to sort data and show relationships (1.2.K2).</li> </ul>	<ul style="list-style-type: none"> <li>i. ▲ frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single and double stem-and-leaf plots, scatter plots, box-and-whisker plots, histograms, and matrices to describe, interpret, and analyze data (2.1.A1, 4.1.A1, 4.1.A3-4, 4.1.A6, 4.2.A1) (<b>\$</b>);</li> <li>j. Venn diagrams to sort data and show relationships.</li> </ul> <p>2. uses the mathematical modeling process to analyze and make inferences about real-world situations (<b>\$</b>).</p>
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**Teacher Notes:** For assessment purposes, the mathematical modeling process appropriate to the indicator may be included as part of the item being assessed.

The **mathematical modeling** process involves:

- a. selecting key features and relationships within the real-world situation and representing these concepts in mathematical terms through some sort of mathematical model,
- b. performing manipulations and mathematical procedures within the mathematical model,
- c. interpreting the results of the manipulations within the mathematical model,
- d. using these results to make inferences about the original real-world situation.

The use of **mathematical models** is necessary for conceptual understanding. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. As students begin to use multiple representations of the same situation, they begin to develop an understanding of the advantages and disadvantages of various representations/models.

Many **mathematical models** are listed in this benchmark. The indicator lists some of the mathematical models that could be used to teach a concept. Each indicator in this benchmark is linked to other indicators in other benchmarks; those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3. In addition, the indicator in the other benchmarks identifies, in parentheses, the Models' indicator. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models).

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (**\$**). The National Standards in Personal Finance are included in the Appendix.

9/10-17  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 3: Geometry**

**NINTH AND TENTH GRADES**

**Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.**

**Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric figures and compares and justifies their properties of geometric figures in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. recognizes and compares properties of two-and three-dimensional figures using concrete objects, constructions, drawings, appropriate terminology, and appropriate technology (2.4.K1h).</li> <li>2. discusses properties of regular polygons related to (2.4.K1g-h):               <ol style="list-style-type: none"> <li>a. angle measures,</li> <li>b. diagonals.</li> </ol> </li> <li>3. recognizes and describes the symmetries (point, line, plane) that exist in three-dimensional figures (2.4.K1h).</li> <li>4. recognizes that similar figures have congruent angles, and their corresponding sides are proportional (2.4.K1h).</li> <li>5. uses the Pythagorean Theorem to (2.4.K1h):               <ol style="list-style-type: none"> <li>a. determine if a triangle is a right triangle,</li> <li>b. find a missing side of a right triangle.</li> </ol> </li> <li>6. recognizes and describes (2.4.K1g-h):               <ol style="list-style-type: none"> <li>a. congruence of triangles using: Side-Side-Side (SSS), Angle-Side-Angle (ASA), Side-Angle-Side (SAS), and Angle-Angle-Side (AAS);</li> <li>b. the ratios of the sides in special right triangles: 30°-60°-90° and 45°-45°-90°.</li> </ol> </li> <li>7. recognizes, describes, and compares the relationships of the angles formed when parallel lines are cut by a transversal (2.4.K1h).</li> <li>8. recognizes and identifies parts of a circle: arcs, chords, sectors of circles, secant and tangent lines, central and inscribed angles (2.4.K1h).</li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. solves real-world problems by (2.4.A1a):               <ol style="list-style-type: none"> <li>a. using the properties of corresponding parts of similar and congruent figures, e.g., scale drawings, map reading, or proportions;</li> <li>b. ▲ ■ applying the Pythagorean Theorem, e.g., when checking for square corners on concrete forms for a foundation, determine if a right angle is formed by using the Pythagorean Theorem;</li> <li>c. using properties of parallel lines, e.g., street intersections.</li> </ol> </li> <li>2. uses deductive reasoning to justify the relationships between the sides of 30°-60°-90° and 45°-45°-90° triangles using the ratios of sides of similar triangles (2.4.A1a).</li> <li>3. understands the concepts of and develops a formal or informal proof through understanding of the difference between a statement verified by proof (theorem) and a statement supported by examples (2.4.A1a).</li> </ol>

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▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes: Geometry** is the study of points, lines, angles, planes, and shapes and their relationships. Symbols and numbers are used to describe the properties of these points, lines, angles, planes, and shapes. The term *geometry* comes from two Greek words meaning “earth measure.” The fundamental concepts in geometry are point (no dimension), line (one-dimensional), plane (two-dimensional), and space (three-dimensional). Plane figures are referred to as two-dimensional and solids are referred to as three-dimensional.

From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), property as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers  $x$  and  $y$ ,  $x + y = y + x$ .
- Property of an equation: For all numbers  $a$ ,  $b$ , and  $c$ , if  $a = b$ , then  $a + c = b + c$ .
- Property of an inequality: For all numbers  $a$ ,  $b$ , and  $c$ , if  $a > b$ , then  $a - c > b - c$ .

The application of the Knowledge Indicators from the Geometry Benchmark, Geometric Figures and Their Properties are most often applied within the context of the other Geometry Benchmarks — Measurement and Estimation, Transformational Geometry, and Geometry From an Algebraic Perspective — rather than in isolation.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-19  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 3: Geometry**

**NINTH AND TENTH GRADES**

**Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.**

**Benchmark 2: Measurement and Estimation – The student estimates, measures and uses geometric formulas in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. determines and uses real number approximations (estimations) for length, width, weight, volume, temperature, time, distance, perimeter, area, surface area, and angle measurement using standard and nonstandard units of measure (2.4.K1a) (\$).</li> <li>2. selects and uses measurement tools, units of measure, and level of precision appropriate for a given situation to find accurate real number representations for length, weight, volume, temperature, time, distance, area, surface area, mass, midpoint, and angle measurements (2.4.K1a) (\$).</li> <li>3. approximates conversions between customary and metric systems given the conversion unit or formula (2.4.K1a).</li> <li>4. states, recognizes, and applies formulas for (2.4.K1h) (\$):             <ol style="list-style-type: none"> <li>a. perimeter and area of squares, rectangle, and triangles;</li> <li>b. circumference and area of circles; volume of rectangular solids.</li> </ol> </li> <li>5. uses given measurement formulas to find perimeter, area, volume, and surface area of two- and three-dimensional figures (regular and irregular) (2.4.K1h).</li> <li>6. recognizes and applies properties of corresponding parts of similar and congruent figures to find measurements of missing sides (2.4.K1a).</li> <li>7. knows, explains, and uses ratios and proportions to describe rates of change (2.4.K1d) (\$), e.g., miles per gallon, meters per second, calories per ounce, or rise over run.</li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. solves real-world problems by (2.4.A1a) (\$):             <ol style="list-style-type: none"> <li>a. converting within the customary and the metric systems, e.g., Marti and Ginger are making a huge batch of cookies and so they are multiplying their favorite recipe quite a few times. They find that they need 45 tablespoons of liquid. To the nearest <math>\frac{1}{4}</math> of a cup, how many cups would be needed?</li> <li>b. finding the perimeter and the area of circles, squares, rectangles, triangles, parallelograms, and trapezoids, e.g., a track is made up of a rectangle with dimensions 100 meters by 50 meters with semicircles at each end (having a diameter of 50 meters). What is the distance of one lap around the inside lane of the track?</li> <li>c. finding the volume and the surface area of rectangular solids and cylinders, e.g., a car engine has 6 cylinders. Each cylinder has a height of 8.4 cm and a diameter of 8.8 cm. What is the total volume of the cylinders?</li> <li>d. using the Pythagorean theorem, e.g., a baseball diamond is a square with 90 feet between each base. What is the approximate distance from home plate to second base?</li> <li>e. using rates of change, e.g., the equation <math>w = -52 + 1.6t</math> can be used to approximate the wind chill temperatures for a wind speed of 40 mph. Find the wind chill temperature (<math>w</math>) when the actual temperature (<math>t</math>) is 32 degrees. What part of the equation represents the rate of change?</li> </ol> </li> <li>2. estimates to check whether or not measurements or calculations for length, weight, volume, temperature, time, distance, perimeter, area, surface area, and angle measurement in real-world problems are reasonable and adjusts original measurement or estimation based on additional information (a frame of reference) (2.4.A1a) (\$).</li> </ol>

9/10-20  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

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|  | <p>3. uses indirect measurements to measure inaccessible objects (2.4.A1a), e.g., you are standing next to the railroad tracks and a train passes. The number of cars in the train can be determined if you know how long it takes for one car to pass and the length of time the whole train takes to pass you.</p> |
|--|--|

**Teacher Notes:** The term *geometry* comes from two Greek words meaning “earth measure.” **Measurement** provides the tools required to apply geometric concepts in the real-world. **Estimation in measurement** is defined as making guesses as to the exact measurement of an object without using any type of measurement tool. Estimation helps students develop a relationship between the different sizes of units of measure. It helps students develop basic properties of measurement and it gives students a tool to determine if a given measurement is reasonable.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-21  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 3: Geometry**

**NINTH AND TENTH GRADES**

**Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.**

**Benchmark 3: Transformational Geometry – The student recognizes and applies transformations on two- and three-dimensional figures in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>describes and performs single and multiple transformations [reflection, rotation, translation, reduction (contraction/shrinking), enlargement (magnification/growing)] on two- and three-dimensional figures (2.4.K1a).</li> <li>recognizes a three-dimensional figure created by rotating a simple two-dimensional figure around a fixed line (2.4.K1a), e.g., a rectangle rotated about one of its edges generates a cylinder; an isosceles triangle rotated about a fixed line that runs from the vertex to the midpoint of its base generates a cone.</li> <li>generates a two-dimensional representation of a three-dimensional figure (2.4.K1a).</li> <li>determines where and how an object or a shape can be tessellated using single or multiple transformations and creates a tessellation (2.4.K1a).</li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>▲ analyzes the impact of transformations on the perimeter and area of circles, rectangles, and triangles and volume of rectangular prisms and cylinders (2.4.A1f), e.g., reducing by a factor of <math>\frac{1}{2}</math> multiplies an area by a factor of <math>\frac{1}{4}</math> and multiplies the volume by a factor of <math>\frac{1}{8}</math>, whereas, rotating a geometric figure does not change perimeter or area.</li> <li>describes and draws a simple three-dimensional shape after undergoing one specified transformation without using concrete objects to perform the transformation (2.4.A1a).</li> <li>uses a variety of scales to view and analyze two- and three-dimensional figures (2.4.A1g).</li> <li>analyzes and explains transformations using such things as sketches and coordinate systems (2.4.A1a).</li> </ol>

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▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Teacher Notes: Transformational geometry** is another way to investigate and interpret geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology. Some **transformations**, like translations, reflections, and rotations, do not change the figure itself. Other transformations, like reduction (contraction/shrinking) or enlargement (magnification/growing), change the size of a figure, but not the shape (congruence vs. similarity).

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-23  
January 31, 2004

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**Standard 3: Geometry**

**NINTH AND TENTH GRADES**

**Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.**

**Benchmark 4: Geometry from an Algebraic Perspective – The student uses an algebraic perspective to analyze the geometry of two- and three-dimensional figures in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>recognizes and examines two- and three-dimensional figures and their attributes including the graphs of functions on a coordinate plane using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or other appropriate technology (2.4.K1f).</li> <li>determines if a given point lies on the graph of a given line or parabola without graphing and justifies the answer (2.4.K1f).</li> <li>calculates the slope of a line from a list of ordered pairs on the line and explains how the graph of the line is related to its slope (2.4.K1f).</li> <li>▲ finds and explains the relationship between the slopes of parallel and perpendicular lines (2.4.K1f), e.g., the equation of a line <math>2x + 3y = 12</math>. The slope of this line is <math>2/3</math>. What is the slope of a line perpendicular to this line? Write an equation for a line perpendicular to <math>2x + 3y = 12</math> (or for multiple choice: Which is an equation of a line perpendicular to <math>2x + 3y = 12</math>?)</li> <li>uses the Pythagorean Theorem to find distance (may use the distance formula) (2.4.K1f).</li> <li>▲ recognizes the equation of a line and transforms the equation into slope-intercept form in order to identify the slope and y-intercept and uses this information to graph the line (2.4.K1f).</li> <li>recognizes the equation <math>y = ax^2 + c</math> as a parabola; represents and identifies characteristics of the parabola including opens upward or opens downward, steepness (wide/narrow), the vertex, maximum and minimum values, and line of symmetry; and sketches the graph of the parabola (2.4.K1f).</li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>represents, generates, and/or solves real-world problems that involve distance and two-dimensional geometric figures including parabolas in the form <math>ax^2 + c</math> (2.4.A1e), e.g., compare the heights of 2 different objects whose paths are represented <math>h_1(t) = 3t^2 + 1</math> and <math>h_2(t) = \frac{1}{2}t^2 + 4</math> (where h represents the height in feet and t represents elapsed time in seconds) after 5 seconds.</li> <li>translates between the written, numeric, algebraic, and geometric representations of a real-world problem (2.4.A1a-e) (\$), e.g., given a situation, write a function rule, make a T-table of the algebraic relationship, and graph the order pairs.</li> <li>recognizes and explains the effects of scale changes on the appearance of the graph of an equation involving a line or parabola (2.4.A1g).</li> <li>analyzes how changes in the constants and/or leading coefficients within the equation of a line or parabola affects the appearance of the graph of the equation (2.4.A1e).</li> </ol>

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January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

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8. explains the relationship between the solution(s) to systems of equations and systems of inequalities in two unknowns and their corresponding graphs (2.4.K1f), e.g., for equations, the lines intersect in either one point, no points, or infinite points; and for inequalities, all points in double-shaded areas are solutions for both inequalities.

**Teacher Notes:** A **coordinate plane** (coordinate grid) consists of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis. These two lines intersect at a point called the origin. The *x*-axis and the *y*-axis divide the plane into four sections called quadrants. Any point on the coordinate plane can be named with two numbers called coordinates. The first number is the *x*-coordinate. The second number is the *y*-coordinate. Since the pair is always named in order (first *x*, then *y*), it is called an ordered pair.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

9/10-25  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 4: Data**

**NINTH AND TENTH GRADES**

**Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.**

**Benchmark 1: Probability – The student applies probability theory to draw conclusions, generate convincing arguments, make predictions and decisions, and analyze decisions including the use of concrete objects in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>1. finds the probability of two independent events in an experiment, simulation, or situation (2.4.K1k) <b>(\$)</b> .</li> <li>2. finds the conditional probability of two dependent events in an experiment, simulation, or situation (2.4.K1k).</li> <li>3. <b>▲</b> explains the relationship between probability and odds and computes one given the other (2.4.K1a,k).</li> </ol>	<p><b>The student...</b></p> <ol style="list-style-type: none"> <li>1. conducts an experiment or simulation with two dependent events; records the results in charts, tables, or graphs; and uses the results to generate convincing arguments, draw conclusions and make predictions (2.4.A1h-i).</li> <li>2. uses theoretical or empirical probability of a simple or compound event composed of two or more simple, independent events to make predictions and analyze decisions about real-world situations including:               <ol style="list-style-type: none"> <li>a. work in economics, quality control, genetics, meteorology, and other areas of science (2.4.A1a);</li> <li>b. games (2.4.A1a);</li> <li>c. situations involving geometric models, e.g., spinners or dartboards (2.4.A1f).</li> </ol> </li> <li>3. compares theoretical probability (expected results) with empirical probability (experimental results) of two independent and/or dependent events and understands that the larger the sample size, the greater the likelihood that experimental results will match theoretical probability (2.4.A1h).</li> <li>4. uses conditional probabilities of two dependent events in an experiment, simulation, or situation to make predictions and analyze decisions.</li> </ol>

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January 31, 2004

**▲ – Assessed Indicator**

**■ – Assessed Indicator on the Optional Response Assessment**

**N – Noncalculator**

**(\$) – Financial Literacy**

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**Teacher Notes:** Ideas from probability reinforce concepts in the other Standards, especially Number and Computation and Geometry. Students need to develop an intuitive concept of chance – whether or not something is unlikely or likely to happen. Probability experiences should be addressed through the use of concrete objects (process models); spinners, number cubes, or dartboards (geometric models); and coins (money models). Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results or outcomes of experiments. Some examples of uses of probability in every day life include: There is a 50% chance of rain today. What is the probability that the team will win every game?

Odds is a ratio of favorable to unfavorable outcomes, whereas probability is a ratio of favorable outcomes to all possible outcomes. If the odds of rolling an even number on a number cube is 1:1 (1/1 or 1 to 1), then the probability is 1/2 (.5 or 50%).

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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9/10-27  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

**Standard 4: Data**

**NINTH AND TENTH GRADES**

**Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.**

**Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (rational) and non-numerical data sets in a variety of situations.**

Ninth and Tenth Grades Knowledge Base Indicators	Ninth and Tenth Grades Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> <li>1. organizes, displays, and reads quantitative (numerical) and qualitative (non-numerical) data in a clear, organized, and accurate manner including a title, labels, categories, and rational number intervals using these <b>data displays</b> (2.4.K1l).               <ol style="list-style-type: none"> <li>a. frequency tables;</li> <li>b. bar, line, and circle graphs;</li> <li>c. Venn diagrams or other pictorial displays;</li> <li>d. charts and tables;</li> <li>e. stem-and-leaf plots (single and double);</li> <li>f. scatter plots;</li> <li>g. box-and-whiskers plots;</li> <li>h. histograms.</li> </ol> </li> <li>2. explains how the reader’s bias, measurement errors, and display distortions can affect the interpretation of data.</li> <li>3. calculates and explains the meaning of range, quartiles and interquartile range for a real number data set (2.4.K1a).</li> <li>4. ▲ explains the effects of outliers on the measures of central tendency (mean, median, mode) and range and interquartile range of a real number data set (2.4.K1a).</li> <li>5. ▲ approximates a line of best fit given a scatter plot and makes predictions using the graph or the equation of that line (2.4.K1k).</li> <li>6. compares and contrasts the dispersion of two given sets of data in terms of range and the shape of the distribution including (2.4.K1k):               <ol style="list-style-type: none"> <li>a. symmetrical (including normal),</li> <li>b. skew (left or right),</li> <li>c. bimodal,</li> <li>d. uniform (rectangular).</li> </ol> </li> </ol>	<p>The student...</p> <ol style="list-style-type: none"> <li>1. ▲ uses data analysis (mean, median, mode, range, quartile, interquartile range) in real-world problems with rational number data sets to compare and contrast two sets of data, to make accurate inferences and predictions, to analyze decisions, and to develop convincing arguments from these <b>data displays</b> (2.4.A1i) (<b>\$</b>):               <ol style="list-style-type: none"> <li>a. ■ frequency tables;</li> <li>b. bar, line, and circle graphs;</li> <li>c. Venn diagrams or other pictorial displays;</li> <li>d. charts and tables;</li> <li>e. stem-and-leaf plots (single and double);</li> <li>f. scatter plots</li> <li>g. box-and-whiskers plots;</li> <li>h. histograms.</li> </ol> </li> <li>2. determines and describes appropriate data collection techniques (observations, surveys, or interviews) and sampling techniques (random sampling, samples of convenience, biased sampling, census of total population, or purposeful sampling) in a given situation.</li> <li>3. uses changes in scales, intervals, and categories to help support a particular interpretation of the data (2.4.A1i).</li> <li>4. determines and explains the advantages and disadvantages of using each measure of central tendency and the range to describe a data set (2.4.K1i).</li> <li>5. analyzes the effects of: outliers on the mean, median, and range of a real number data set; changes within a real number data set on mean, median, mode, range, quartiles, and interquartile range.</li> </ol>

9/10-28  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

6. approximates a line of best fit given a scatter plot, makes predictions, and analyzes decisions using the equation of that line (2.4.A1i).

**Teacher Notes: Graphs (data displays)** are pictorial representations of mathematical relationships and are an important part of statistics. When a graph is made, the axes and the scale (numbers running along a side of the graph) are chosen for a reason. The difference between numbers from one grid line to another is the interval. The interval will depend on the lowest and highest values in the data. Emphasizing the importance of using equal-sized pictures or intervals is critical to ensuring that the data is displayed accurately.

Graphs take many forms:

- bar graphs compare discrete data;
- frequency tables show how many times a certain piece of data occurs;
- circle graphs (pie charts) model parts of a whole;
- line graphs show change over time;
- Venn diagrams show relationships between sets of objects;
- line plots show frequency of data on a number line;
- stem-and-leaf plots (closely related to line plots) show frequency distribution by arranging numbers (stems) on the left side of a vertical line (single stem-and-leaf) with numbers (leaves) on the right side;
- scatter plots show the relationship between two quantities;
- box-and-whisker plots are visual representations of the five-number summary – the median, the upper and lower quartiles, and the least and greatest values in the distribution – therefore, the center, the spread, and the overall range are immediately evident by looking at the plot; and
- histograms (closely related to stem-and-leaf plots) describe how data falls into different ranges.

Two important aspects of data are its *center* and its *spread*. The mean, median, and mode are measures of central tendency (averages) that describe where data are centered. Each of these measures is a single number that describes the data. However, each does it slightly differently. The **range** describes the spread (dispersion) of data. The easiest way to measure spread is the range, the difference between the greatest and the least values in a data set. Quartiles are boundaries that break the data into fourths. The interquartile range is the difference between the upper quartile and the lower quartile and is considered a measure of dispersion.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

9/10-29  
January 31, 2004

▲ – Assessed Indicator

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N – Noncalculator

(§) – Financial Literacy

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The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **(\$)**. The National Standards in Personal Finance are included in the Appendix.

9/10-30  
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Response Assessment

N – Noncalculator

(\$) – Financial Literacy

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